SAMPLE MATH 55 MIDTERM 1, SPRING 2014

(1) Mark each of the following questions true (T) or false (F). Provide a sentence or two justifying each answer.

(a) If \( x \equiv y \pmod{m} \) then \( ax \equiv ay \pmod{m} \).

(b) If \( ax \equiv ay \pmod{m} \) then \( x \equiv y \pmod{m} \).

(c) The function \( f : \mathbb{Z} \to \mathbb{Z} \) defined by \( f(x) = \lfloor \frac{x}{2} \rfloor \) is surjective.

(d) \( f(S \cap T) = f(S) \cap f(T) \).

(e) The positive real numbers are countable.

(f) Let \( \mathbb{R} \) be the domain, and let \( P(x, y) \) be the statement \( y^2 = x \). Determine the truth value of the following statement: \( \forall x \exists y P(x, y) \).
(2) Prove that if $m$ is a positive integer of the form $4k + 3$ for some non-negative integer $k$, then $m$ is not the sum of the squares of two integers.
(3) Prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$. 
(4) Computation.
   - Write the number 466 in base 9.

   - Does an inverse of 8 (mod 75) exist? If so, find one.

   - Calculate $6^{666} \mod 23$. 
(5) Prove that if \( p \) is prime, the only solutions of \( x^2 \equiv 1 \pmod{p} \) are integers \( x \) such that \( x \equiv 1 \pmod{p} \) or \( x \equiv -1 \pmod{p} \).
(6) Find all solutions to the system of congruences $x \equiv 2 \pmod{3}$, $x \equiv 1 \pmod{4}$, and $x \equiv 3 \pmod{5}$. 