Math 53 Midterm #2, 4/10/07, 3:40 PM – 5:00 PM
(please do not leave the exam between 4:45 and 5:00)

No calculators or notes are permitted. Each of the 6 questions is worth 10 points. Please write your solution to each of the 6 questions on a separate sheet of paper with your name, SID number, and GSI’s name on it. To get full credit, you must put a box around your final answer and show correct work/justification. Good luck!

1. Find the volume of the solid region between the surfaces \( z = 2x^2 + 2y^2 \) and \( z = 12 - x^2 - y^2 \).

2. Find the minimum and maximum values of the function
   \[ f(x, y) = x^2 + y^2 + 5y \]
   on the region \( x^2 + y^2 \leq 4 \), and say where the function takes these values.

3. Evaluate the iterated integral
   \[ \int_0^1 \int_x^1 \frac{\cos y}{y} \, dy \, dx. \]

4. Evaluate the triple integral
   \[ \iiint_E (x^2 + y^2 + z^2)^{3/2} \, dV, \]
   where \( E \) is the region determined by the inequalities \( x^2 + y^2 + z^2 \leq 1 \), \( z \geq 0 \), and \( z^2 \leq x^2 + y^2 \).

5. Let \( R \) denote the triangle in the \( x, y \) plane with corners at \((0, 0), (1, 0), \) and \((0, 1)\). Use the change of variables \( x = u^2, y = v^2 \) to evaluate the double integral
   \[ \iint_R \frac{1}{\sqrt{xy}} \, dA. \]

6. Evaluate the iterated integral
   \[ \int_0^{1/\sqrt{2}} \int_x^{\sqrt{1-x^2}} e^{x^2 + y^2} \, dy \, dx. \]