In this exam you may assume, without justification the following identity:

\[ \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e \]
1. Determine if the following sequences converge or diverge. Carefully justify your answer.
   
   (a) (10 points)
   \[ \left\{ \frac{e^{-n}}{\sin \left(\frac{1}{n}\right)} \right\}_{n=1}^{\infty} \]
   
   Solution:

   (b) (10 points)
   \[ \left\{ \frac{1}{2 + (-1)^n} \right\}_{n=1}^{\infty} \]
   
   Solution:
2. (20 points) Using the integral test, prove the following series is convergent

\[ \sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}. \]

Using this, prove that

\[ \sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^3 + 1} \]

is convergent.

Solution:
3. (20 points) Determine if the following series is convergent or divergent

\[ \sum_{n=1}^{\infty} \frac{(n - 1)2\sin(n^2)}{n^4 + 3n + 1} \]

**Solution:**
4. Determine whether the following series are convergent or divergent. If convergent determine the sum.

(a) (10 points)
\[ \sum_{n=1}^{\infty} n \tan\left(\frac{1}{n}\right) \]

Solution:

(b) (10 points)
\[ \sum_{n=1}^{\infty} \frac{10^n + 5^n}{6^n + 4^n + 3^n} \]

Solution:
5. (20 points) Determine whether the following series is absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^5}{\sin(\frac{1}{n})n!}$$

**Solution:**