# MATH H1B – Exam 2 Proofs Preview

***Part I (typical)*: Monday, October 31, 10am-11am**
**(one notecard/ half-sheet of paper allowed)**

***Part II (proofs)*: Tuesday, November 1, 11am-12:30pm**

At least one of the following proofs will appear on Part II of the exam (it will be a surprise which one/ones). The variable names and/or constants might be changed, but the content of the proof will be exactly as stated below.

For all proofs, you may use any of our basic axioms and theorems about arithmetic without specifically quoting them. As usual, proofs should be in paragraph form.

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RULES: You are allowed to consult your class notes, any handouts from the course website, your textbook, and each other to figure these out, but NO OTHER RESOURCES. Michael and I will only answer questions if it is unclear what the question is asking, and you are not permitted to talk to anyone outside the class about the problems, or to consult other books or anything on the internet other than our course website. On Monday, you will sign an honor code indicating that you have followed these rules.

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**Candidate 1.** State and prove Rolle's Theorem. (You may use Fermat's Theorem from the book, provided you can clearly state it.)

**Candidate 2.** Using a figure and 1-2 paragraphs, summarize the proof of the Mean Value Theorem (assuming Rolle's Theorem has already been proved).

**Candidate 3.** Prove that if a power series $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = b$ (where $b \neq 0$), then it converges whenever $|x| < |b|$.

**Candidate 4.** Prove the following special case of the Limit Comparison Theorem. Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. Prove that if $\sum b_n$ is divergent and $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$, then $\sum a_n$ is divergent. (Note: we have a precise definition of what it means for the limit of a sequence to be infinity; use it! You can find it in Section 11.1.)

**Candidate 5.** Let

$$f(x) = \begin{cases} 2x^2 & x \neq 5 \\ 1 & x = 5. \end{cases}$$

Find $\lim_{x\to 3} f(x)$ and give an $\epsilon$-$\delta$ proof that your answer is correct. Is $f(x)$ continuous at $x = 3$? Justify your answer using the rigorous definition.

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**Some notes about homework:**

Homework #6 scores were not so great – make sure you discuss these problems with each other!

On Homework #7, I did not carefully grade your series testing (Problem 1); I did check to make sure you were doing a reasonable amount of work of the right type, so pretty much everyone got a 10 on the first problem.

I did not yet look at some of the challenge problems. If you have challenge problems with no comments yet, please bring those on Tuesday, and I'll give some feedback on your work.

Only one person gave me old homework for scanning, so I am taking that to mean most of you don't care about getting nice solutions up online...