MATH H1B EXAM #1, PART II

Tuesday, September 27, 2011

No calculators are permitted on this portion of the exam. You may use one card of notes with dimensions 5 inches by 7 inches or smaller.

You must show your work for all problems, unless specifically indicated otherwise. Answers with insufficient or incorrect work will be given little (if any) credit. You are not required to write all your explanations in complete sentences, but your work should be organized and easy to read. You may use the back of any page to do scratchwork that you don't want graded. If you make a mistake, you can cleanly erase or put an X or a single line through anything we should ignore. (No crazy scribbling, please.)

Some advice: don't stay stuck on a problem for long unless you have already solved the other problems. I arranged the problems roughly in order of increasing difficulty, and I don't expect that everyone will finish everything.

This exam has 6 problems on 9 pages, including this cover sheet.

Problem	Max	Score
1	18	
2	16	
3	15	
4	12	
5	14	
6	15	
Total	90	

1. (3 points each) Determine whether each of the statements below is true or false. Choose TRUE only if the statement is always true under the given circumstances. No justification is required for this problem.

   (a) Suppose f and g are continuous functions such that $f(x) \geq g(x) \geq 0$ for all x in the interval $[a, b]$. Then the length of the curve f between $x = a$ and $x = b$ is greater than or equal to the length of the curve g between $x = a$ and $x = b$.

   TRUE *FALSE*

   (b) Suppose p and q are continuous functions such that $p(x) \geq q(x) \geq 0$ for all $x \in \mathbb{R}$. Then $\int_0^c p(x)dx \geq \int_0^c q(x)dx$ for all $c \in \mathbb{R}$.

   TRUE *FALSE*

   (c) The partial fraction decomposition of $\frac{x^4+x^3+x^2}{(x^2+1)^2(x-2)}$ is of the form

   $$\frac{A}{x^2+1} + \frac{B}{(x^2+1)^2} + \frac{C}{x-2},$$

   where A, B, and C are real numbers.

   TRUE *FALSE*

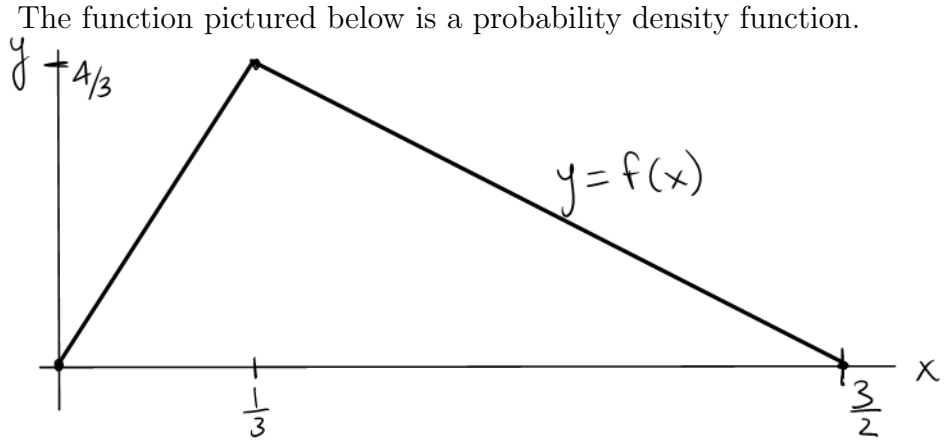
   (d) If $\int_0^\infty h(x)dx$ and $\int_0^\infty k(x)dx$ are both convergent, then $\int_0^\infty (h(x) + k(x))dx$ is also convergent.

   TRUE *FALSE*

   (e) If $\int_0^\infty |g(x)|dx$ diverges and $|f(x)| \leq |g(x)|$ for all x, then $\int_0^\infty f(x)dx$ converges.

   TRUE *FALSE*

   (f) The function pictured below is a probability density function.



   TRUE *FALSE*

2. Let T be the flat triangular plate whose vertices are $(0,0)$, $(2,6)$, and $(2,-4)$. The density of the plate is given by the function $\rho = 3(x+2)$.

   (a) (4 points) Find the mass of the plate. Be sure to include a figure and show what happens on a slice.

   (b) (4 points) Find the x-coordinate of the centroid of T.

This page is a continuation of Problem 2.

(c) (4 points) Thinking about the y-coordinate of the centroid of any appropriately chosen slice of T, you can show that the centroid of T must lie on a particular line. What is an equation for this line?

(d) (4 points) Using parts (b) and (c), find the y coordinate of the centroid of T.

3. Using a rope, it takes 2 minutes to lift a 50-pound box of dirt from the ground to a height of 20 feet above the ground. As the box is lifted, dirt falls out of the box at a constant rate such that a total of 8 pounds of dirt has been lost when it reaches the final height of 20 feet. If you need to use the constant for gravity in this problem, just leave it as "g", rather than filling in $g = 9.8$.

   (a) (5 points) Let h be the height, in feet, of the box above the ground. Write an expression estimating the work done in raising the box from a height of h_i feet to a height of $h_i + \Delta h$ feet, ignoring the weight of the rope. Somewhere on the page, there should be a figure.

   (b) (5 points) Find the total amount of work done in raising the box of dirt from the ground to the final height of 20 feet, ignoring the weight of the rope.

   (c) (5 points) Suppose the rope lifting the box of dirt weighs 0.5 pounds per foot and dangles from a platform that is 30 feet above the ground. Find the total work done to lift the box of dirt to a height of 20 feet above the ground, taking into account the weight of the rope.

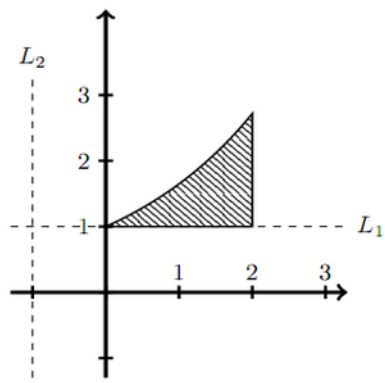
4. (12 points) Let t be the number of minutes a student waits for the Perimeter bus. For some constants a and b, the probability density function giving the distribution of t is

$$p(t) = \begin{cases} 0 & \text{if } x < 0 \\ ae^{-bt} & \text{if } 0 \leq x. \end{cases}$$

Assume that the mean waiting time for the bus is 8 minutes. Using this fact and what you know about probability density functions, find the exact values of the constants a and b.

$a =$ _____ $b =$ _____

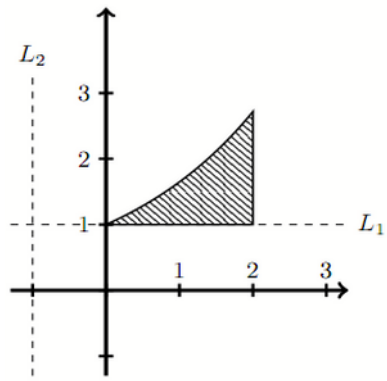
5. The graph of the region bounded by the curve $y = e^{0.5x}$, the line $y = 1$, and the line $x = 2$ is pictured below. We will find the surface area of the solid formed by rotating this region around the line L_2, in several steps. *(You may use basic geometry facts to do parts (a) and (b).)*



(a) (3 points) Find the exact value of SA_{side}, the surface area of the vertical side of the surface.

(b) (3 points) Find the exact value of SA_{bottom}, the surface area of the horizontal bottom of the surface.

This page is a continuation of Problem 5. The figure is repeated for your convenience.



(c) (8 points) Write a definite integral which is SA_{top}, the surface area of the curved top of the surface. Do NOT evaluate the integral. It is not mandatory to show what happens on a slice for this problem, but you may find it helpful.

6. (15 points) Suppose an ocean sunfish is swimming in salt water with a density of 1050 kg/m^3. Approximate the fish using the thin plate shown below (the plate consists of a circle and an isosceles triangle, and it is sitting vertically in the water), with the center of the circle at a depth of 20 m. (Ocean sunfish can get quite big!)

   Choose an appropriate coordinate system and write a sum of 2-3 definite integrals which gives the total hydrostatic force on (our idealized version of) the ocean sunfish. Be sure your work shows what happens on a slice of each region you consider. Do NOT evaluate the integral.